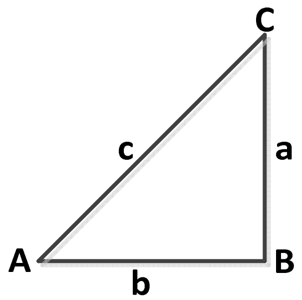
（09）三角函數方程式

三角函數有很多方程式，同學們應該知道這些方程式都是很容易證明的。

（1）試證 $\boldsymbol{sin^2(\theta) + cos^2(\theta) = 1}$

見下圖:



$sin(\theta) = \frac{a}{c}$

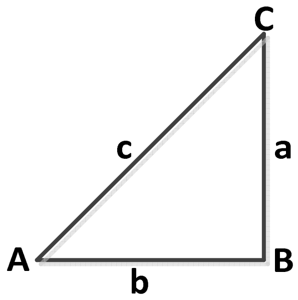
$cos(\theta) = \frac{b}{c}$

故 1

$$\boldsymbol{sin^2(\theta) + cos^2(\theta)}$$
$$= \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$$
$$= \frac{a^2}{c^2} + \frac{b^2}{c^2}$$
$$= \frac{a^2 + b^2}{c^2}$$
$$= \frac{c^2}{c^2}$$
$$= 1$$

(2)試證 $1 + tan^2(\theta) = sec^2(\theta)$

請看下圖:



$$1 + tan^2(\theta)$$
$$= 1 + \frac{a^2}{b^2}$$
$$= \frac{a^2 + b^2}{b^2}$$
$$= \frac{c^2}{b^2}$$
$$= sec^2(\theta)$$

以下是同學們該記得的三角公式，而且要能導出這些公式:

$$\boldsymbol{sin(90\,^{\circ} - \theta) = cos(\theta)}$$
$$\boldsymbol{cos(90\,^{\circ} - \theta) = sin(\theta)}$$
$$\boldsymbol{tan(90\,^{\circ} - \theta) = cot(\theta)}$$

$$\boldsymbol{tan(\theta) = \frac{sin(\theta}{cos(\theta)}}$$

$$\boldsymbol{sin^2(\theta) + cos^2(\theta) = 1}$$
$$\boldsymbol{1 + tan^2(\theta) = sec^2(\theta)}$$

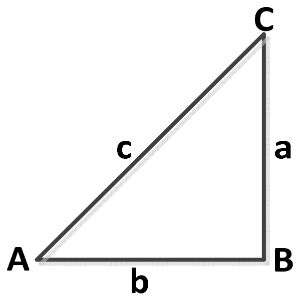
(3) 已知$\boldsymbol{sin(\theta) = \frac{1}{2}}$，求$\boldsymbol{cos(\theta)}$

方法 1

$$sin^2(\theta) + cos^2(\theta) = 1$$
$$\frac{1}{4} + cos^2(\theta) = 1$$
$$cos^2(\theta) = 1 - \frac{1}{4} = \frac{3}{4}$$

$cos(\theta) = \frac{\sqrt{3}}{2}$

方法 2



$sin(\theta) = \frac{1}{2} = \frac{a}{c}$

$a = \frac{1}{2}c$

$b^2 = c^2 - a^2 = c^2 - \frac{1}{4}c^2 = \frac{3}{4}c^2$

$b = \frac{\sqrt{3}}{2}c$

$cos(\theta) = \frac{b}{c} = \frac{\sqrt{3}}{2}$

(4) 已知$tan(\theta) = \frac{3}{4}$, 求$cos(\theta)$

$1 + tan^2(\theta) = sec^2(\theta)$

$sec^2(\theta) = 1 + tan^2(\theta) = 1 + \left(\frac{3}{4}\right)^2 = 1 + \frac{9}{16} = \frac{25}{16}$

$sec(\theta) = \frac{5}{4}$

$cos(\theta) = \frac{4}{5}$